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THE PROPAGATION OF GEOMAGNETIC MICRO-PULSATIONS INTO A TWO-LAYER CONDUCTING EARTH

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Geomagnetic micropulsation fields at the earth's surface are characterized by spatial variations along the surface that are associated with non-uniform plane waves in the Fourier spectrum representation of these near fields. Using both a single micropulsation frequency and a single spatial frequency in a two-layer conducting earth, expressions are derived for the field attenuation, the ratio of vertical to horizontal field, and the surface impedance. Numerical results are presented for a seawater layer over an earth bottom for a spatial frequency that is prominent in many micropulsations.

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INTRODUCTION

Geomagnetic micropulsations at the surface of the earth at frequencies well below 1 Hz may conventionally be explained as being the near field effects of electric currents produced by decaying hydromagnetic waves in the lower ionosphere. The spatial variation of the electromagnetic field along the earth's surface will depend not only on the frequency of the micropulsations and the conductivity structure below the surface but also on the spatial extent of the ionospheric currents.

Using Fourier methods, the curved, near field wavefronts associated with these geomagnetic micropulsations may be synthesized from a spectrum of plane waves that includes both uniform plane waves (planes of constant phase coincident with planes of constant amplitude) and non-uniform plane waves (non-coincident planes of constant phase and constant amplitude). Because of the very high index of refraction of the earth at frequencies below 1 Hz, a uniform plane wave incident on a plane earth propagates essentially vertically into the earth and the vertical field components virtually vanish. For example, in a semi-infinite conducting medium of seawater, the ratio of the vertical to the horizontal magnetic field is less than 10^{-7} for a frequency of 10^{-3} Hz. Also, because of the very large free space wavelengths at frequencies below 1 Hz, the spatial variation of the field along the earth's surface is practically negligible at all angles of incidence and the surface is essentially a plane of constant phase. In the case of a non-uniform plane wave spectral component, the surface of the earth is a plane of constant amplitude and the planes of constant phase in free space are normal to the earth's surface. This field geometry can result in a significant spatial variation of the electromagnetic field along the surface and the spatial periodicity of this variation is in general unrelated to the free space wavelength. For a horizontally layered earth this horizontal spatial variation must be the same for all the layers in order to satisfy the boundary conditions at each interface.

It is the purpose of this report to elucidate the effect of the horizontal spatial variation of the electromagnetic field on

(1) the attenuation of both the horizontal and vertical components of geomagnetic micropulsations in a two-layer medium with plane interfaces

(2) the ratio of the vertical to horizontal magnetic field at the top surface of such a medium

(3) the surface impedance of a two-layer medium.

Numerical results are presented for a seawater layer over a conducting earth bottom.

Raff (1960) and Price (1965) have addressed themselves to the same problem; however, in some respects their treatments are not as general. Raff besides choosing a rather unrealistic spatial variation parameter does not consider the ratio of the vertical to horizontal magnetic field components and furthermore presents incorrect numerical results for the attenuation of the vertical magnetic field. Price considers only the case of a non-conducting bottom, which besides being unrealistic, greatly exaggerates the effects of the horizontal spatial variation of the field.

FORM OF PLANE WAVES IN LAYERED MEDIA

A detailed treatment of electromagnetic waves in stratified media is available in Wait (1962). Some important results in this treatment are summarized in Kraichman (1970).

For the purposes of the present report, only the transverse electric field (TE) case will be considered since this case is applicable for sources consisting of horizontal current sheets in the ionosphere. Thus for a general, single spectral component plane wave incident on a horizontally stratified conducting medium in which the top surface coincides with the X-Y plane, the Z axis points into the layered medium, and the current in the ionosphere flows parallel to the Y-axis, the electric field in the mth layer is given by

$$E_{my} = \left(a_m e^{-u_m z} + b_m e^{u_m z}\right) e^{-j\nu x} \tag{1}$$

where $u_m{}^2 = \nu^2 + \gamma_m{}^2$, $\gamma_m{}^2 = -\omega^2\mu_m\varepsilon_m + j\omega\mu_m\sigma_m$ with the real part of $\gamma_m > 0$, and ν can take any real value. A time variation $e^{j\omega t}$ is assumed. In (1) the amplitudes of the downward and upward traveling plane waves in the mth layer are a_m and b_m respectively, σ_m is the conductivity of the layer, and μ_m and ε_m are its permeability and permittivity respectively.

The magnetic field components in the mth layer are

$$H_{mx} = \frac{1}{j\omega\mu_m}\frac{\partial E_{my}}{\partial z} = \frac{u_m}{j\omega\mu_m}\left(-a_m e^{-u_m z} + b_m e^{u_m z}\right) e^{-j\nu x} \tag{2}$$

$$H_{mz} = -\frac{1}{j\omega\mu_m}\frac{\partial E_{my}}{\partial x} = \frac{\nu}{\omega\mu_m}\left(a_m e^{-u_m z} + b_m e^{u_m z}\right) e^{-j\nu x}$$

$$= \frac{\nu}{\omega\mu_m} E_{my} \tag{3}$$

Since the spatial variation along the X-axis is the same for all the layers, the propagation of the horizontal components of the electromagnetic field in the layers may be determined by using the well-known transmission line analogy which is based on the one-to-one correspondence between reflection coefficient at an interface and the impedance mismatch ratio.

THE ATTENUATION OF PLANE WAVE FIELDS IN A TWO-LAYER MEDIUM

The equations (1)-(3) for the fields when combined with the transmission line analogy readily give the attenuation of the electric and magnetic fields in a two-layer medium. Of particular interest is the ratio of a field quantity at the bottom of the first layer, $z = z_1$, to that field quantity at the surface, $z = 0$, of the first layer. In the event of a stratification of more than 2 layers, the results below will hold if the second layer is assigned an effective impedance that may be calculated from the formulas in Kraichman (1970) for the impedance properties of a layered medium.

Horizontal Magnetic Field Attenuation

It is desired to find the ratio $H_{1x}(z_1)/H_{1x}(0)$. From (2),

$$H_{1x}(z_1) = \frac{u_1}{j\omega\mu_1}\left(-a_1 e^{-u_1 z_1} + a_1\rho_{12}e^{-u_1 z_1}\right)e^{-j\nu x} \quad (4)$$

$$H_{1x}(0) = \frac{u_1}{j\omega\mu_1}\left(-a_1 + a_1\rho_{12}e^{-2u_1 z_1}\right)e^{-j\nu x} \quad (5)$$

where the reflection coefficient at the interface between layers 1 and 2 is

$$\rho_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (6)$$

and the impedances

$$\eta_1 = \frac{j\omega\mu_1}{u_1}$$

$$\eta_2 = \frac{j\omega\mu_2}{u_2}. \quad (7)$$

(It should be noted that the reflection coefficient for the magnetic field is the negative of ρ_{12} defined in (6).) The first term in (4) represents a plane wave of amplitude a_1 traveling through the top layer from $z = 0$ to $z = z_1$ and the second term is the value of this plane wave at $z = z_1$ after reflection at the interface between medium 1 and medium 2. In (5) the first term represents the value of the downward traveling plane wave at $z = 0$ and the second term, which represents the value of the upward traveling plane wave at $z = 0$, is merely the first term after an excursion through the top layer, a reflection at the bottom, and a return trip to $z = 0$. The ratio sought then becomes

$$\frac{H_{1x}(z_1)}{H_{1x}(0)} = \frac{(1 - \rho_{12})e^{-u_1 z_1}}{1 - \rho_{12}e^{-2u_1 z_1}} \tag{8}$$

Vertical Magnetic Field Attenuation

From (3)

$$\frac{H_{1z}(z_1)}{H_{1z}(0)} = \frac{E_{1y}(z_1)}{E_{1y}(0)} = \frac{(1 + \rho_{12})e^{-u_1 z_1}}{1 + \rho_{12}e^{-2u_1 z_1}} \tag{9}$$

using the same reasoning as in (8).

RATIO OF FIELDS AT THE SURFACE OF A TWO-LAYER MEDIUM

It is of interest to derive an expression for the ratio of the vertical to horizontal components of the magnetic field at the top surface, $z = 0$, of a two-layer medium. Using (1)-(3),

$$H_{1z}(0) = \frac{\nu}{\omega\mu_1} E_{1y}(0) = \frac{\nu}{\omega\mu_1}(a_1 + b_1)e^{-j\nu x} \tag{10}$$

$$H_{1x}(0) = \frac{u_1}{j\omega\mu_1}(-a_1 + b_1)e^{-j\nu x} \tag{11}$$

so that

$$\frac{H_{1z}(0)}{H_{1x}(0)} = \frac{j\nu}{u_1}\left(\frac{1 + \frac{b_1}{a_1}}{-1 + \frac{b_1}{a_1}}\right) \tag{12}$$

Since at z = 0

$$\frac{\nu}{\omega\mu_1} b_1 e^{-j\nu x} = \frac{\nu}{\omega\mu_1} a_1 \rho_{12} e^{-2u_1 z_1} e^{-j\nu x}$$

it follows that

$$\frac{b_1}{a_1} = \rho_{12} e^{-2u_1 z_1} \tag{13}$$

so that

$$\frac{H_{1z}(0)}{H_{1x}(0)} = \frac{j\nu}{u_1}\left(\frac{1 + \rho_{12}e^{-2u_1 z_1}}{-1 + \rho_{12}e^{-2u_1 z_1}}\right) \tag{14}$$

It is also of interest to derive an expression for the surface impedance, η_s, of a two-layer medium. This impedance is defined as the ratio $E_{1y}(0)/H_{1x}(0)$. Using (1) and (2) and the value of b_1/a_1 in (13),

$$\eta_s = \frac{j\omega\mu_1}{u_1}\left(\frac{1 + \frac{b_1}{a_1}}{-1 + \frac{b_1}{a_1}}\right)$$

$$= \frac{j\omega\mu_1}{u_1}\left(\frac{1 + \rho_{12}e^{-2u_1 z_1}}{-1 + \rho_{12}e^{-2u_1 z_1}}\right) \tag{15}$$

VALUES OF THE SPATIAL PROPAGATION CONSTANT ν

A non-periodic spatial variation of the field along the surface of the earth may be synthesized by a superposition of sinusoidal components in which the spatial propagation constant ν takes on all values from 0 to ∞. The behavior of each sinusoidal component is described by the foregoing equations in which a spatial wavelength λ_s may be associated with each value of ν such that $\nu = 2\pi/\lambda_s$. The value $\nu \to 0$ or $\lambda_s \to \infty$ describes the field due to a current sheet of infinite width and implies a normally incident uniform plane wave.

Ionospheric currents of finite extent produce an aperiodic spatial variation of the field along the earth's surface. The effects of this variation could in principle be obtained by means of Fourier transforms; however, such a procedure is not only complex mathematically, but also depends on an accurate knowledge of the aperiodic surface variation which is generally not available. It is often possible to get a reasonable approximation of the effects by applying the equations derived above to a prominent sinusoidal component of the spatial variation if the spatial wavelength of such a component can be estimated. Since in most cases it is impractical to determine the spatial variation of the fields, an attempt will be made to arrive at a range of values for ν that will represent spatial harmonics that are prominent in many geomagnetic fluctuations and which can give reasonable estimates for the field attenuations and ratios described earlier.

Although the plane surface model is used in deriving the previous equations for the two-layer medium, the fact that the radius of the earth is so much larger than any depth of the top conducting layer considered and is also larger than the skin depth in the bottom layer lends some validity to this approximation. This is especially so for horizontal spatial variations which are not completely global in extent. The in-phase ionospheric currents are most likely limited to an earth quadrant and a prominent spatial wavelength, λ_s, along the earth's surface is then of the order of 2 quadrants. Thus, a lower limit for ν may reasonably be taken as

$$\nu = \frac{2\pi}{\lambda_s} \simeq 3 \times 10^{-7} m^{-1}$$

An upper limit for ν may be estimated from the spatial field variation along the earth's surface due to an extremely localized source such as a line current. This spatial variation on the surface of a semi-infinite conducting earth ($\sigma = 1.25 \times 10^{-4}$ mho/m) has been calculated by Law and Fannin (1961) for a line current at a frequency of approximately 0.05 Hz located at a height of 200 km in the ionosphere. A prominent spatial frequency associated with field variations along the earth's surface may be estimated from the graphical results presented by these authors. Thus the half sinusoid that most nearly fits the field variations in the vicinity of the line source has a spatial wavelength $\lambda_s = 1.8 \times 10^6$ m and a corresponding $\nu = 2\pi/\lambda_s = 3.6 \times 10^{-6}$ m^{-1}. From the foregoing, the values of the spatial propagation constant ν may reasonably be taken to lie between the limits

$$3 \times 10^{-7} < \nu < 3.5 \times 10^{-6} \qquad (16)$$

As a demonstration of the practical value of this approach, the maximum value of $|H_{1z}(0)/H_{1x}(0)|$, which is a sensitive function of ν, is seen from the graphical results of Law and Fannin to be equal to 0.48 and occurs at a surface distance of 400 km from the line current source. Using the above value of $\nu = 3.5 \times 10^{-6}$ m^{-1} in (14) for the semi-infinite medium yields 0.51 for this ratio.

NUMERICAL RESULTS FOR THE TWO-LAYER MEDIUM

Numerical results are presented here for a seawater layer of conductivity σ_1 = 4 mho/m and a depth z_1 = 200 meters over a semi-infinite earth bottom of conductivities $\sigma_2 = 4 \times 10^{-2}$ mho/m and 4×10^{-4} mho/m. All layers are assumed to have the permeability of free space. Oscillation frequencies of 10^{-2}, 10^{-3}, and 10^{-4} Hz are chosen for the ionospheric current source. The value of $\nu = 10^{-6}$ m^{-1} is used for the spatial propagation constant and corresponds to approximating the spatial field variation by a half wavelength equal to 3×10^6 meters or about 1900 miles. Such an intermediate value for ν should be representative of a goodly number of geomagnetic micropulsations.

Whether or not the horizontal spatial variation will affect the horizontal and vertical field attenuations given by (8) and (9) spectively will depend on what contribution ν makes to the values of u_1 and u_2 which are defined by

$$u_1 = (\gamma_1{}^2 + \nu^2)^{1/2} \tag{17}$$

$$u_2 = (\gamma_2{}^2 + \nu^2)^{1/2} \tag{18}$$

where $\gamma_1{}^2 \approx j\omega\mu_o\sigma_1$ and $\gamma_2{}^2 \approx j\omega\mu_o\sigma_2$. Table 1 gives the ratios $|\nu^2/\gamma_1{}^2|$ and $|\nu^2/\gamma_2{}^2|$ for various combinations of the two-layer model parameters described above.

Table 1. Ratios $|\nu^2/\gamma_1{}^2|$ and $|\nu^2/\gamma_2{}^2|$ for Various Two-Layer Model Parameters with $\nu = 10^{-6}$ m^{-1}

		10^{-2} Hz	10^{-3} Hz	10^{-4} Hz
$\left\lvert\frac{\nu^2}{\gamma_1{}^2}\right\rvert$	σ_1 = 4 mho/m	3.2×10^{-6}	3.2×10^{-5}	3.2×10^{-4}
$\left\lvert\frac{\nu^2}{\gamma_2{}^2}\right\rvert$	σ_2 = 4×10^{-2} mho/m	3.2×10^{-4}	3.2×10^{-3}	3.2×10^{-2}
	σ_2 = 4×10^{-4} mho/m	3.2×10^{-2}	0.32	3.2

It may be seen from Table 1 that the contribution of ν is quite small for all combinations of the parameters chosen except for the frequencies 10^{-4} Hz and 10^{-3} Hz and a bottom conductivity of 4 x 10^{-4} mho/m.

For the case where the frequency is 10^{-4} Hz and σ_2 = 4 x 10^{-4} mho/m, taking $\nu = 10^{-6}$ m^{-1} rather than $\nu = 0$ results in a 52% increase in the horizontal magnetic field attenuation ratio given by (8). For the other significant case where the frequency is 10^{-3} Hz and σ_2 = 4 x 10^{-4} mho/m, the increase is only 0.4%. All other combinations in Table 1 yield essentially the same value for the horizontal magnetic field attenuation ratio as that for uniform plane wave incidence ($\nu \approx 0$). The vertical magnetic field (or horizontal electric field) attenuation ratio given by (9) is virtually unaffected even for those cases in Table 1 in which ν makes a significant contribution to u_2.

The ratio of the vertical to the horizontal magnetic field at the surface of a two-layer medium is a rather sensitive function of ν since the expression in (14) contains a factor that depends on ν directly. These ratios are presented in Table 2 for the two-layer model parameters described earlier.

Table 2. Ratio $|H_{1z}(0)/H_{1x}(0)|$ for Various Two-Layer Model Parameters with z_1 = 200 meters and $\nu = 10^{-6}$ m^{-1}

	10^{-2} Hz	10^{-3} Hz	10^{-4} Hz
σ_2 = 4 x 10^{-2} mho/m	.0091	.04[illegible]	.16
σ_2 = 4 x 10^{-4} mho/m	.015	.13	.75

It is evident from (14) and Table 2 that for a given bottom conductivity the ratio $|H_{1z}(0)/H_{1x}(0)|$ approaches unity as the frequency decreases.

A significant change in the surface impedance due to the horizontal spatial variation occurs only for the case in Table 1 in which the frequency is 10^{-4} Hz and the bottom conductivity is 4 x 10^{-4} mho/m. Using (15), the effect of taking $\nu = 10^{-6}$ m^{-1} rather than $\nu = 0$ results in a 16% decrease in the surface impedance.

SUMMARY AND CONCLUSIONS

Spatial variations of geomagnetic micropulsation fields along the surface of a horizontally stratified conducting earth are the result of near field effects produced by the ionospheric current sources of the micropulsations. These spatial variations are associated with the non-uniform plane waves in the Fourier spectrum representation of the near field.

Assuming a two-layer, horizontally stratified, conducting medium and using both a single micropulsation frequency and a single spatial frequency, expressions are derived for the horizontal and vertical magnetic field attenuation to the bottom of the top layer, the ratio of the vertical to horizontal magnetic field at the surface of the two-layer medium, and the surface impedance of this medium. Limits are obtained for the prominent spatial wavelengths associated with geomagnetic micropulsation activity. Using a spatial wavelength that is most likely prominent in many micropulsations, numerical results are presented for a seawater layer over an earth bottom.

Assigning a value of 10^{-6} m^{-1} for the spatial propagation constant and the values 4×10^{-2} and 4×10^{-4} mho/m for the bottom conductivity, results are obtained for micropulsation frequencies of 10^{-2}; 10^{-3}, and 10^{-4} Hz in a seawater layer 200 meters deep. These results show that only for the lowest value of bottom conductivity and micropulsation frequency does the spatial variation along the sea surface contribute significantly to the attenuation of the horizontal magnetic field to the bottom. The effect is to increase the attenuation by 52 percent. All other combinations of micropulsation frequencies and bottom conductivities yield essentially the same value as that for uniform plane wave incidence ($\nu \approx 0$). The vertical magnetic field (or horizontal electric field) attenuation to the bottom is virtually unaffected for all combinations of the parameters above.

Only in the case of the ratio of the horizontal to vertical magnetic field at the surface of the above two-layer medium does the spatial variation along the surface have a large influence. In fact, in the absence of conductivity inhomogeneities other than the assumed horizontal stratification, it is this spatial variation that largely determines the value of this ratio which for very low micropulsation frequencies and bottom conductivities can approach unity. It should be noted that the value of this ratio at the sea bottom is greater than its value at the surface since the vertical magnetic field attenuation to the bottom is less than that for the horizontal magnetic field.

As is the case for the horizontal magnetic field attenuation, a significant change in the surface impedance due to the horizontal spatial variation occurs only for the lowest value assumed for the micropulsation frequency and bottom conductivity and results in a 16% decrease in the surface impedance.

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